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Unfair allocation of gains under equal price in cooperative purchasing

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Abstract / Summary

Cooperative purchasing is becoming more and more common practice. However, many cooperative initiatives end prematurely or do not flourish. Important reasons indicated for these problems are directly or indirectly related to the unfair allocation of gains. The purpose of this paper is to analyse causes of unfairness in current cooperative practices, and in particular unfairness resulting from using the Equal Price allocation concept. I suggest that the unfair effects of this commonly used concept are caused by neglecting a specific part of the added value of cooperative initiative members. Moreover, I prove that when using the Equal Price concept organisations will receive fewer gains if they increase their volume past 38% of the total volume of a cooperative initiative. In case of a constant total volume I prove that Equal Price reaches its maximum pay-off when the volume of an organisation equals 25%. I conclude by emphasizing the importance of cooperative members becoming aware of allocation concept problems. Further research will involve possible solutions to these problems.

Key words

Cooperative purchasing; allocation of gains; equal price; cooperative game theory.

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Introduction

Cooperative purchasing initiatives as purchasing consortia, purchasing groups, and buying offices are not a new idea (Hendrick, 1997). Studies of cooperative purchasing go back as far as 1927 (Mitchell). Ever since, many definitions have been used for cooperative purchasing. In this paper cooperative purchasing is defined as *the sharing or bundling of purchasing related information, experiences, processes, resources or volumes to improve the performance of all participating organisations*.

Cooperative purchasing has received relatively little attention in purchasing management research. In addition, cooperative purchasing research so far has focused primarily on inductive explanations of practice and qualitative deductive reasoning (Laing, 1997; Mudambi, 2004). The use of quantitative deductive reasoning has been limited until now (Essig, 1998; Heijboer, 2003). The lack of research attention seems unjustified, with cooperative purchasing being more and more well-established (Doucette, 1997; Macie, 1995; Major, 1997; Sickinger, 1996; Zentes, 2000).

One specific issue receiving little research attention is the allocation of the direct financial gains resulting from cooperative purchasing. Important reasons indicated for cooperative purchasing problems – dealing with differences in size, anti-trust, no commitment and ‘fear of parasites’ – are related to the allocation of gains (Heijboer, 2003; Schotanus, 2004).

Usually, purchasing consortia use the so-called Equal Price (EP) allocation concept for allocating price savings obtained by pooling their purchasing power: *all organisations pay the same price per item*. While practically appealing, EP may lead to unfair outcomes. For instance, the situation could occur that an organisation increases its purchases through the initiative, but in return receives a smaller amount of the total gains. This could slow down potential growth and harm the stability of the cooperative initiative.

This has been reported previously by Heijboer (2003), but a systematic analysis is still lacking. Furthermore, it seems that many consortia using EP are unaware of its potential unfairness. To this end an in-depth survey was carried out as a foundation for this paper (Schotanus, 2004). All cooperative initiatives in this survey used EP. Most initiatives (73%) indicated not being aware of all possible unfairness effects of EP. The actual financial gains are on average indicated as being the most important reason to purchase cooperatively. Other studies confirm these results (Aylesworth, 2003).

This paper provides an analytical analysis of unfair outcomes of using EP, provides recommendations for purchasing consortia as how to deal with it, and contributes to more awareness and understanding of the problem. In a more general sense this paper aims to contribute to the quantitative deductive development of purchasing management. The main questions in this paper are: (1) how does EP lead to unfair outcomes, and (2) which circumstances determine the extent of unfairness?

The organisation of the paper is as follows. First, I develop a formal model of cooperative purchasing that enables analysing unfairness effects while using EP. In the next section I use the model to investigate what exactly makes EP result in unfair outcomes. I do this by decomposing the added value of a consortium into three components and study how applying EP affects each component separately. In the following section I study how the *degree* of unfairness is affected by the relative stake of each consortium member. In the final sections I discuss the limitations of the research, draw conclusions, and provide recommendations for purchasing consortia and scholars in the field.

A cooperative purchasing model

To analyse the effects of the Equal Price (EP) allocation concept, I model cooperative purchasing initiatives by taking into account price reduction due to economies of scale (Heijboer, 2003). Of course other issues also play a role in the success of establishing and managing cooperative initiatives. Here I focus on the actual cooperative financial gains as this is indicated as being an important reason to purchase cooperatively. Furthermore, quantity discounts of some products may be very dependent to i.e. individual transportation costs, decreasing the direct cooperative financial gains. These items are left out of the focus.

For the price per item $p(q)$ I assume a decreasing volume discount is given, with more items being purchased. Of course there is a minimum price p_0 , so $p(q)$ is a convex function. This gives the demand elasticity of price: *the change in percentage in the price resulting from a change in percentage in the quantity demanded* (Ramsay, 1981).

In addition, I assume the total purchasing volume $q \cdot p(q)$ to be increasing with the number of items being bought. These assumptions hold for many practical situations (Dolan, 1987; Melymuka, 2001). This model is defined as a Cooperative Purchasing-game or CP-game(N, q, p) (Heijboer, 2003). N is the number of organisations, q is the number of items each organisation i wants to purchase and p is the price per item. The total gains function $v(S)$ of each coalition S is defined as *the gains it generates by buying items together compared to the situation where each of the organisations has to buy these items on its own*:

$$v(S) = \sum_{i \in S} (q_i \cdot p(q_i)) - \sum_{i \in S} q_i \cdot p\left(\sum_{i \in S} q_i\right) \quad (1)$$

The model builds on cooperative game theory. In cooperative game theory it is assumed that gains can be made when all players cooperate. One of the problems that are addressed in this theory is how to divide these gains in a fair way among all players. Each of the players should receive a fair part of the total gains (Dyer, 2000).

Unfairness illustrated

With the following case I will illustrate the gain allocation effects of current practices in cooperative purchasing. Consider 3 organisations purchasing 60 items cooperatively. The price p for the items as a function of the quantity q that will be ordered is known as:

$$p(q_i) = p_0 \cdot \left(c_1 + \frac{c_2}{\sqrt{q_i}}\right) = 959 \cdot \left(1 + \frac{1}{\sqrt{q_i}}\right) \text{ for } q_i > 0 \quad (2)$$

Here p_0 represents the minimum price, c_1 and c_2 are used to further shape and scale the function. The three organisations use the EP-concept: all organisations pay the price that can be obtained with the volume of the grand coalition N . This case can be modelled into a CP-game as is shown in Table 1.

Table 1: CP-game for three organisations

| <i>Coalition S</i> | <i>Quantity</i> | <i>Price per item</i> | <i>Total</i> | <i>v(S)</i> |
|--------------------|-----------------|-----------------------|---------------|--------------|
| {1} | 35 | 1.121 | 39.246 | 0 |
| {2} | 10 | 1.262 | 12.625 | 0 |
| {3} | 15 | 1.207 | 18.102 | 0 |
| {1,2} | 45 | 1.102 | 49.597 | 2.273 |
| {1,3} | 50 | 1.095 | 54.741 | 2.607 |
| {2,3} | 25 | 1.151 | 28.775 | 1.952 |
| {1,2,3}=N | 60 | 1.083 | 64.980 | 4.993 |

Now the gains that each case organisation receives when the cooperative initiative {1,2,3} uses the EP-concept can be calculated and analysed:

Organisation i gains: $q_i \cdot (p(q_i) - p(q_N))$ while using EP

Organisation 1 gains: $35 \cdot (1.121 - 1.083) = \mathbf{1.341}$ (*largest organisation*)

Organisation 2 gains: $10 \cdot (1.262 - 1.083) = \mathbf{1.795}$ (*smallest organisation*)

Organisation 3 gains: $15 \cdot (1.207 - 1.083) = \mathbf{1.857}$

Total gains: $1.341 + 1.795 + 1.857 = \mathbf{4.993}$

A remarkable outcome is that using EP leads to a situation where the largest organisation receives the smallest part of the total gains. The smallest organisation however receives the largest part of the total gains. The largest organisation could object to this allocation, as the largest organisation adds the most value to the cooperative initiative. The smallest organisation adds the least value, as will be shown in the next section. Such an unfair situation could lead to instability in the cooperative initiative, because the largest player could leave the initiative or could lower its commitment.

How does Equal Price lead to unfairness?

In this section I extend the model to investigate the underlying mechanism that causes EP to produce unfair outcomes. I do this by formally defining the added value of consortia, breaking it down into three components, and studying the impact of EP on each component.

The added value of a purchasing consortium

In real life situations organisations can add value in several ways to a cooperative initiative. Here, the added value, or in other words the total gains an organisation creates for the cooperative initiative, is defined as *the total gains of the coalition minus the value the other organisations can establish without organisation i* (Borm, 1992):

$$M_i(v) = v(N) - v(N \setminus \{i\}) \quad (3)$$

Note that in the model the larger the organisation is, the more value this organisation adds to the initiative. Given $M_i(v)$ the added value of the case organisations 1 and 2 is:

Organisation 1 Added Value: $4.993 - 1.952 = \mathbf{3.041}$ (*largest organisation*)

Organisation 2 Added Value: $4.993 - 2.607 = \mathbf{2.386}$ (*smallest organisation*)

To obtain more insight into the added value, I split this value into three different parts: (1) gains for and by an organisation created by joining a cooperative initiative (m_i), (2) gains

created by an organisation for the other organisations in the initiative (n_i) and, (3) gains for an organisation created by the other organisations in the initiative (o_i). The added value M_I of i.e. *case organisation 1* can be divided into these three types of gains as is shown in Table 2.

Table 2: gains of organisation 1

| Gains | Description | Calculation | Total |
|--|--|------------------------------|---------------|
| m_i = gains for and by 1 | $= q_i \cdot \left(\min_{S:i \in S} \left\{ p\left(\sum_{j \in S/i} q_j\right), p(q_i) \right\} - p\left(\sum_{i \in N} q_i\right) \right)$ | $= 35 \cdot (1.121 - 1.083)$ | $= 1.341$ |
| n_i = gains by 1 for $N \setminus \{1\}$ | $= \sum_{j \in S/i} q_j \cdot \left(p\left(\sum_{j \in S/i} q_j\right) - p\left(\sum_{j \in S} q_j\right) \right)$ | $= 25 \cdot (1.151 - 1.083)$ | $= 1.700$ |
| o_i = gains for 1 by $N \setminus \{1\}$ | $= \max_{S:i \in S} \left\{ q_i \cdot \left(p(q_i) - p\left(\sum_{j \in S/i} q_j\right) \right), 0 \right\}$ | $= 35 \cdot (1.121 - 1.121)$ | $= 0$ |
| Total | maximum claim of 1 | =M_I | =3.041 |

Equal Price neglects one component of the added value

The EP-concept neglects one component of the added value as is shown in theorem and proof 1:

Theorem 1

The Equal Price concept neglects n_i .

Proof

$$\begin{aligned}
 equalprice_i(v) &= q_i \cdot \left(p(q_i) - p\left(\sum_{i \in N} q_i\right) \right) \\
 &= \begin{cases} q_i \cdot \left(p(q_i) - p\left(\sum_{i \in N} q_i\right) \right) & p\left(\sum_{j \in S/i} q_j\right) \leq p(q_i) \\ q_i \cdot \left(p(q_i) - p\left(\sum_{i \in N} q_i\right) \right) & p\left(\sum_{j \in S/i} q_j\right) > p(q_i) \end{cases} \\
 &= \begin{cases} q_i \cdot \left(p(q_i) + p\left(\sum_{j \in S/i} q_j\right) - p\left(\sum_{j \in S/i} q_j\right) - p\left(\sum_{i \in N} q_i\right) \right) & p\left(\sum_{j \in S/i} q_j\right) \leq p(q_i) \\ q_i \cdot \left(p(q_i) - p\left(\sum_{i \in N} q_i\right) \right) + 0 & p\left(\sum_{j \in S/i} q_j\right) > p(q_i) \end{cases} \\
 &= q_i \cdot \left(\min_{S:i \in S} \left\{ p\left(\sum_{j \in S/i} q_j\right), p(q_i) \right\} - p\left(\sum_{i \in N} q_i\right) \right) + \max_{S:i \in S} \left\{ q_i \cdot \left(p(q_i) - p\left(\sum_{j \in S/i} q_j\right) \right), 0 \right\} \\
 &= m_i + o_i
 \end{aligned}$$

Because n_i is neglected, EP is unfair in situations where organisations differ significantly in size:

Organisation 1 Equal Price: ($m_1 + o_1$) = **1.341** (largest organisation)
Organisation 2 Equal Price: ($m_2 + o_2$) = **1.795** (smallest organisation)
Organisation 1 Added Value: ($m_1 + o_1$) + n_1 = 1.341 + 1.700 = **3.041**
Organisation 2 Added Value: ($m_2 + o_2$) + n_2 = 1.795 + 591 = **2.386**

Relationship between the degree of unfairness in the outcomes and presence of relatively large members and general rules

The impact of member size on m_i , n_i , and o_i

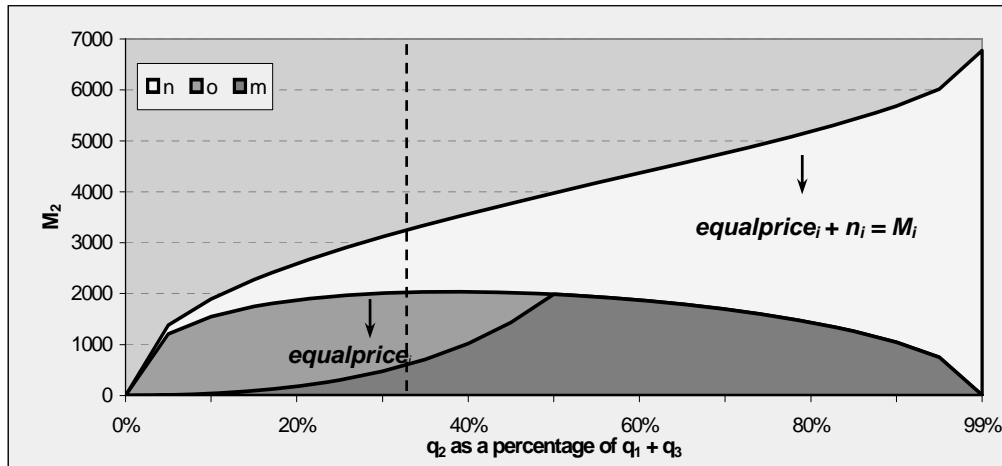
In the model there are three disadvantages to the EP-concept which apply especially to large members. First, n_i is always increasing with more items being purchased by *organisation i*. Therefore it becomes less attractive for larger cooperative members to use the EP-concept. After all, n_i is not incorporated in this concept, and the larger the value of n_i , the more these members are put at a disadvantage.

The second disadvantage is that o_i becomes 0 after a certain point. This point is independent of the price structure (p_0 , c_1 , c_2), the number of organisations, and the division of the volumes of these organisations. The point where o_i becomes 0 is always reached when the volume of *organisation i* equals exactly 50% of the total volume of the initiative. In such a case a *large organisation i* using the EP-concept receives just the value of $m_i + o_i = m_i + 0 = m_i$. The maximum value of o_i is always reached when the volume of *organisation i* equals exactly 20% of the total volume. This part of the o_i -disadvantage applies therefore to organisations purchasing more than 20% of the total cooperative volume. In this case larger organisations receive a relatively small part of o_i . Proofs have been omitted here.

Thirdly, past the point where the volume of *large organisation i* exceeds 50% of the total cooperative volume, m_i will become smaller by an increasing volume of *organisation i*. At least past this point the total EP-outcomes always decrease - as o_i is already 0 - even if this organisation increases its volume through the initiative.

The 38% rule

Figure 1: Type of gains for organisation 2 with different quantities of q_2 while $q_{1+3} = 50$ is constant



Theorem 2

While using the EP concept and given the price function (2), consortium members purchasing more than 38% of the total volume are always put at a disadvantage; they will receive fewer gains with an increasing volume.

Proof

$$equalprice_i(v) = q_i \cdot (price(q_i) - price(N))$$

$$= q_i \cdot (p_0 \cdot (c_1 + \frac{c_2}{\sqrt{q_i}}) - p_0 \cdot (c_1 + \frac{c_2}{\sqrt{N}}))$$

$$= p_0 \cdot c_2 \cdot \sqrt{q_i} - \frac{p_0 \cdot c_2 \cdot q_i}{\sqrt{N}}$$

Here, $N = q_i + N \setminus q_i$ and,

$$equalprice_i(v)' = \frac{p_0 \cdot c_2}{2\sqrt{q_i}} - \frac{p_0 \cdot c_2}{\sqrt{N}} + \frac{p_0 \cdot c_2 \cdot q_i}{2N^{1.5}} = 0$$

$$\frac{p_0 \cdot c_2}{2\sqrt{q_i}} - \frac{p_0 \cdot c_2}{\sqrt{N}} + \frac{p_0 \cdot c_2 \cdot q_i}{2N^{1.5}} = 0$$

$$\frac{1}{2\sqrt{q_i}} - \frac{1}{\sqrt{N}} + \frac{q_i}{2N^{1.5}} = 0$$

$$\text{if } q_i = FFR \cdot N \text{ then } \frac{1}{2\sqrt{FFR \cdot N}} - \frac{1}{\sqrt{N}} + \frac{FFR \cdot N}{2N^{1.5}} = 0$$

$$FFR + \frac{1}{\sqrt{FFR}} - 2 = 0$$

$$FFR = \frac{3 - \sqrt{5}}{2} = 38 \%$$

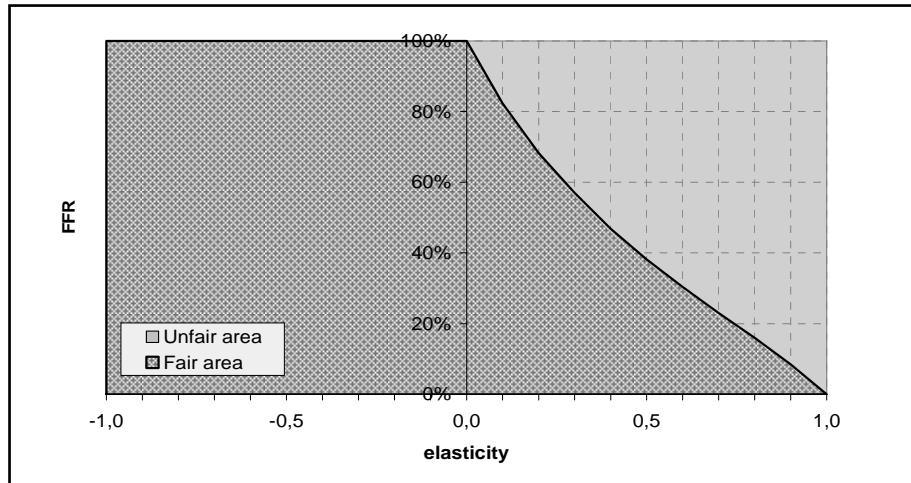
The only dependent variable in this proof is η in the following continuous price function:

$$p(q_i) = p_0 \cdot (c_1 + \frac{c_2}{q_i^\eta}) \text{ for } q_i > 0 \quad (4)$$

η represents the elasticity of the price function. Until now I assumed η always being 0,5. However, 0,5 is an estimated average value (Dolan, 1987) and in practice η may vary. For values of η between -1 and 1 the following function applies (see also Figure 2):

$$\eta \cdot FFR^{1+\eta} - FFR^\eta - \eta + 1 = 0 \quad (5)$$

Figure 2: Dependency of FFR to η
(for all η the maximum added value is reached by $q=100\%$)

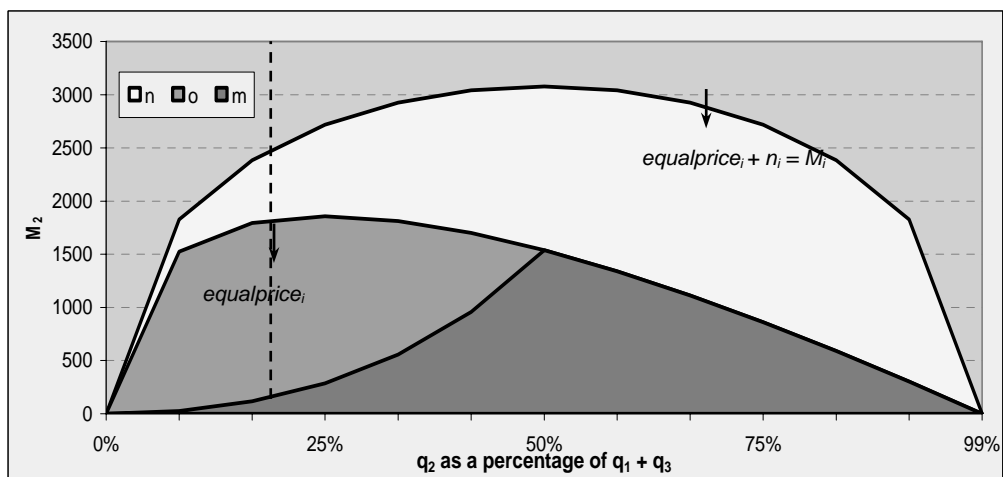


With an elasticity of 1,0 and a corresponding FFR of 0%, all organisations increasing their volume through the consortium will receive fewer gains. The smallest organisation will always receive the largest part of the gains. The largest organisation will always receive the smallest part of the gains.

The 25% rule

The 38% rule applies to organisations increasing or decreasing their cooperative volume. Now I consider the situation where the total volume of a consortium is fixed. Figure 3 illustrates this scenario for different quantities of *organisation 2*.

Figure 3: Type of gains for organisation 2 with different quantities of q_2 while N is constant



At the point where q_2 becomes 50% of the total volume, the added value of this organisation reaches its maximum value. At the point where q_2 becomes 25% of the total volume, the EP-outcome for *organisation 2* already reaches its maximum. With theorem and proof 3 I prove that this is always the case in the model. Again, this percentage is independent of p_o , c_1 and c_2 in the price structure, the number of organisations, and the division of the volumes of these organisations.

To conclude, when using EP and assuming a continuous price function, organisations purchasing 25% of the total volume will receive the maximum allocation of gains. Larger or smaller members will receive a smaller amount of gains. I define 25% as the Second Fairness Ratio (SFR) of EP with an average price function.

Theorem 3

While using the EP concept and given the price function (2), consortium members purchasing 25% of the total volume will receive the maximum allocation of gains.

Proof

$$\begin{aligned} equalprice_i(v) &= q_i \cdot (price(q_i) - price(N)) \\ &= p_0 \cdot c_2 \cdot \sqrt{q_i} - \frac{p_0 \cdot c_2 \cdot q_i}{\sqrt{N}} \end{aligned}$$

Here, $N = \text{fixed}$ and,

$$equalprice_i(v)' = \frac{p_0 \cdot c_2}{2\sqrt{q_i}} - \frac{p_0 \cdot c_2}{\sqrt{N}} = 0$$

$$\frac{1}{2\sqrt{q_i}} - \frac{1}{\sqrt{N}} = 0$$

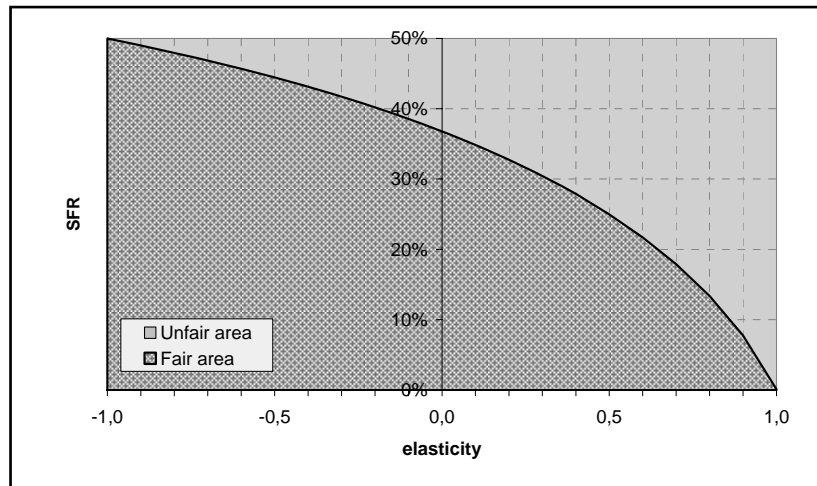
$$q_i = \frac{N}{4}$$

if $q_i = SFR \cdot N$ then $SFR = 25\%$

Once more, the dependent variable in this proof is η . If $\eta = -1$, $SFR = 50\%$. This is a fair situation as SFR equals the point where the added value reaches its maximum (50%). When $\eta > -1$, $SFR < 50\%$, this could lead to an unfair situation as SFR reaches its maximum before the added value does. For SFR the following function applies (see also Figure 4):

$$SFR = (1 - \eta)^{\frac{1}{\eta}} \leq FFR \tag{6}$$

Figure 4: Dependency of SFR to η
(for all η the maximum added value is reached by $q=50\%$)



Limitations and further research

Before I draw conclusions I point out the main limitations of the research that should be taken into account. First, modelling continuous price functions is a simplification of reality. In reality usually graduated prices are used when quantity discounts apply (Munson, 1998). For instance, a price of 4.000 applies to 50-99 items, and a price of 3.900 applies to 100-199 items. However, when an organisation needs 98 items it will usually negotiate a lower price than 4.000, or otherwise it will order 100 items. Therefore, I use a continuous price function instead of a graduated price function. All different forms of graduated prices (Dolan, 1987) can be approximated by a continuous price function. Other researchers also proposed the existence of continuous price functions (i.e. Dolan, 1987; Jeuland, 1983; Spence, 1977).

Secondly, I do not take into account the costs of cooperating and other advantages than financial gains. This could compensate unfairness effects. Furthermore, in some cases a smaller company may be able to negotiate a lower price than a larger company. Obviously the suitability of purchasing consortia may be questioned here.

To increase the relevance and applicability of the results, further research will incorporate (1) taking into account cooperative initiative setup and transaction costs, (2) taking more benefits of cooperation into account than just volume discounts and (3) finding solutions to unfairness problems by i.e. taking into account the total added value of partners.

Conclusion

In this paper I show that the Equal Price concept ignores an important part of the added value of organisations. Therefore this concept results in unfair allocations of gains for large members of cooperative initiatives, as buying groups or (electronic) purchasing consortia.

Moreover, I conclude that under the assumption of a convex and continuously differentiable average price function and using Equal Price, organisations increasing their

volume past 38% of the total volume of a cooperative initiative will receive fewer gains. Even though their added value and the total gains of the cooperative initiative increase. Furthermore, I conclude that the Equal Price value always reaches its maximum when the volume of an organisation becomes 25% of the total constant volume of a cooperative initiative. As a result it becomes less attractive for larger organisations to cooperate.

To conclude, if organisations are unequal in size - or size differences among previously similar members increase steadily - and they use the Equal Price concept, it is important that they address this issue in an open manner and develop solutions for it in order to avoid instability of the cooperative initiative on the longer term.

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